Ambiguity and Incomplete Information in Categorical Models of Language

Dan Marsden

June 9, 2016

- Meaning are modelled in dagger compact closed categories, typical examples are **FdVect**, **CPM(FdVect)**, **Rel**
- Meanings of individual words are states in these compact closed categories, e.g. $I \xrightarrow{dog} N$
- How do we describe ambiguous and incomplete information in a systematic way?

Multiple Interpretations

When we encounter the noun “bank” in a sentence, questions arise:

Multiple Interpretations

When we encounter the noun “bank” in a sentence, questions arise:

- Does the text intend a “river bank” or a “financial bank”?

Multiple Interpretations

When we encounter the noun "bank" in a sentence, questions arise:

- Does the text intend a "river bank" or a "financial bank"?
- Is one more likely than the other?

Multiple Interpretations

When we encounter the noun “bank” in a sentence, questions arise:

- Does the text intend a “river bank” or a “financial bank”?
- Is one more likely than the other?
- Given representations of “river bank” and “financial bank” can we build a model of “bank” as a mixture?

$$0.9\,\text{“river bank”} + 0.1\,\text{“financial bank”}$$

- Move to **CPM**(**FdHilb**) and model ambiguity using density matrices:

$$0.9|r.b\rangle\langle r.b| + 0.1|f.b.\rangle\langle f.b|$$

Categorical Compositional Semantics

Ambiguity and Selinger's CPM Construction

- Move to **CPM**(**FdHilb**) and model ambiguity using density matrices:

$$0.9|r.b\rangle\langle r.b| + 0.1|f.b.\rangle\langle f.b|$$

- Can we systematically use the CPM construction to provide a "mixing construction"? **CPM(Rel)** seems to suggest the answer is no:
  - Strange mixing behaviour
  - Poverty of scalars

Categorical Compositional Semantics

Ambiguity and Selinger's CPM Construction

- Move to **CPM**(**FdHilb**) and model ambiguity using density matrices:

$$0.9|r.b\rangle\langle r.b| + 0.1|f.b.\rangle\langle f.b|$$

- Can we systematically use the CPM construction to provide a "mixing construction"? **CPM(Rel)** seems to suggest the answer is no:
  - Strange mixing behaviour
  - Poverty of scalars
- Do we care?

Adding Mixtures Formally

Expanding Homsets

If we start with a dagger compact closed category $\mathcal{C}$, we would like "expand" our homsets with extra elements that represent probabilistic mixtures of the morphisms in $\mathcal{C}$. Ideally:

- Formation of mixtures should interact well with composition - enriched category
- Our new category should contain $\mathcal{C}$ as the "pure" information
- Our new category will also be dagger compact closed
- In the absence of further information, we want to do this "freely", introducing only the required equations

Informational Effects as Algebra

Phenomena of Interest

Probabilistic (quantified) ambiguity:

> *With confidence p it's a "river bank", otherwise it's a "financial bank"*

Modelled as a family of binary operations:

$$f +^p g$$

With axioms:

$$\begin{aligned} f +^p g &= g +^{1-p} f \\ f +^p f &= f \\ f +^p (g +^q h) &= (f +^m g) +^n h \end{aligned}$$

Unquantified ambiguity:

It's either a "river bank" or a "financial bank"

Modelled as a binary operation with:

$$f \vee g$$

Satisfying commutativity, associativity and idempotence. An affine join semilattice.

Informational Effects as Algebra

Phenomena of Interest

Incomplete information:

I don't know what "logolepsy" means

Modelled as a constant:

$$\bot$$

Combinations of ambiguity and incomplete information:

I'm 90% certain "bank" means "financial bank", otherwise I don't know what it refers to

$$\text{“financial bank”} +^{p} \bot$$

The algebras for describing quantitative mixtures are the algebras of the **finite distribution monad** D. They form a category of **convex algebras**, denoted **Convex**. The free algebra on a set F is given by all *formal* convex combinations:

$$\sum_i p_i |f_i\rangle$$

The algebras for describing unquantified mixing are the algebras of the **non-empty finite powerset monad** P_{ω}^{+}. They form a category of **affine join semilattices**, denoted **AJSLat**. The free algebra on a set F is given by all non-empty finite subsets of F.

The algebras for describing incomplete information are the algebras of the **lift monad** $(-)_{\perp}$. They form a category of **pointed sets**, denoted $\mathbf{Set}_{\bullet}$. The free algebra on a set F is given by extending F with a $\perp$ element.

The algebras for describing a combination of quantified ambiguity and incomplete information are the algebras of the **subdistribution monad** S. They form a category of **subconvex algebras**, denoted **Subconvex**. The free algebra on a set F is given is given by all *formal* subconvex combinations:

$$\sum_i p_i |f_i\rangle$$

The algebras for description a combination of unquantified ambiguity and incomplete information are the algebras of the **finite powerset monad** P_ω. They form a category of **join semilattices**, denoted **JSLat**. The free algebra on a set F is given by the set of finite subsets of F.

Categories of Algebras

Using standard results, the Eilenberg-Moore category of a **Set** monad is:

- Complete.
- Cocomplete.

Categories of Algebras

Using standard results, the Eilenberg-Moore category of a **Set** monad is:

- Complete.
- Cocomplete.

In fact, all the monads we're interested in are commutative. In this case (Kock, Jacobs), the Eilenberg-Moore category:

- Is symmetric monoidal closed.
- Has universal bimorphisms for the monoidal tensor.
- Has monoidal unit given by the free algebra $(\{*\}, !)$.
- Has tensors of free algebras given by free algebras on the cartesian product

Informational Effects as Algebras

Enrichments and Composition

A category is:

- **Set$_\bullet$**-enriched if its homsets are pointed sets such that:

$$\bot \circ f = \bot \quad \text{and} \quad f \circ \bot = \bot$$

- **AJSLat**-enriched if its homsets are affine semilattices such that:

$$(f \vee g) \circ h = (f \circ h) \vee (g \circ h) \quad \text{and} \quad f \circ (g \vee h) = (f \circ g) \vee (f \circ h)$$

- **Convex**-enriched if its homsets are convex algebras such that:

$$(\sum_i p_i f_i) \circ g = \sum_i p_i (f_i \circ g) \quad \text{and} \quad f \circ (\sum_i p_i g_i) = \sum_i p_i (f \circ g_i)$$

Results

For a dagger compact closed category $\mathcal{C}$ we can:

- Construct a category $\mathcal{C}_\perp$ who's homsets contain an additional $\perp$ element
- Construct a category $\mathcal{C}_{P_\omega^+}$ with homsets containing non empty subsets of $\mathcal{C}$-morphisms.
- Construct a category $\mathcal{C}_D$ with homsets containing all formal convex sums of $\mathcal{C}$-morphisms.
- Construct a category $\mathcal{C}_{P_\omega}$ with homsets containing finite subsets of $\mathcal{C}$-morphisms.
- Construct a category $\mathcal{C}_S$ with homsets containing all formal subconvex sums of $\mathcal{C}$-morphisms.

Results

In each case we can extend the composition and monoidal structure in $\mathcal{C}$ giving a category such that:

- There is an identity on objects embedding of $\mathcal{C}$ into the new category
- It is the free enrichment of $\mathcal{C}$ with respect to the appropriate algebraic structure.
- The category is dagger compact closed.

Conclusion

- We can "expand" homsets to incorporate various informational effects
- Our extended homsets contain exactly the additional elements that we need
- Even starting with categories such as **Rel** we can form genuine probabilistic mixtures if we need them

Conclusion

- We can "expand" homsets to incorporate various informational effects
- Our extended homsets contain exactly the additional elements that we need
- Even starting with categories such as **Rel** we can form genuine probabilistic mixtures if we need them
- Is it a coincidence that all these monads are commutative?
- Are there other information based phenomena that can be incorporated?
- The general process needs making explicit.