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no-broadcasting in process theories
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...the full story in <15 mins

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no-broadcasting in process theories

...including all math from scratch

Bob Coecke & Aleks Kissinger

_Picturing Quantum Processes._

Cambridge University Press, 2016. (920pp)
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*Categorical quantum mechanics I: causal quantum processes.*

arXiv:1510.05468

*Categorical quantum mechanics II: classical-quantum interaction.*

arXiv:1605.08617

*Categorical quantum mechanics III: (strong) complementarity.*

arXiv:XXX (next couple of weeks)
Diagram := boxes with ins-outs wired together
Process theory := interpretation of diagrams
Process theory := interpretation of diagrams

• assign to each wire a system

• assign to each box a process
Process theory := interpretation of diagrams

- assign to each wire a **system**
- assign to each box a **process**
- interprets ‘wiring together’
Process theory := interpretation of diagrams

• assign to each wire a system

• assign to each box a process

• **interprets ‘wiring together’** e.g.:
  – as actual physical wires,
Process theory := interpretation of diagrams

- assign to each wire a system

- assign to each box a process

- interprets ‘wiring together’ e.g.:
  - as actual physical wires, or,
  - via decomposing in $\circ$ and $\otimes$. 
Special processes:

• **State** :=

• **Effect/Test** :=
Special processes:

- **State** :=

- **Effect/Test** :=

- **Number** :=
Special processes:
String diagram := ins and outs on equal footing
- teleportation -
– transpose –
Adjoint := H-reflection
...conjugate
...conjugate := V-reflection
Main idea:

\[
\frac{\text{classical system}}{\text{quantum system}} = \frac{\text{single wire}}{\text{double wire}}
\]
Pure quantum process :=
Born-rule :=

\[ \hat{\phi} \quad \hat{\psi} \] := \begin{array}{c}
\begin{array}{c}
\phi \\
\psi
\end{array}
\end{array} \quad \begin{array}{c}
\begin{array}{c}
\phi \\
\psi
\end{array}
\end{array}
Discarding :=

\[ \begin{array}{c}
\text{\begin{tikzpicture}
    \draw[thick] (0,0) -- (1,0);
    \draw[thick] (1,0) -- (1,1);
    \draw[thick] (1,1) -- (0,1);
\end{tikzpicture}}
\end{array} = \begin{array}{c}
\text{\begin{tikzpicture}
    \draw[thick] (0,0) -- (1,0);
    \draw[thick] (1,0) -- (1,1);
    \draw[thick] (1,1) -- (0,1);
    \draw[thick, dashed] (0,1) -- (1,2);
\end{tikzpicture}}
\end{array} \]
General quantum process :=

\[
\hat{f} = \begin{array}{c}
\hline
\hline
f
\end{array}
\]

\[
\begin{array}{c}
\hline
\hline
f
\hline
\hline
f
\end{array}
\]
THM. No universal broadcasting
THM. No universal broadcasting i.e. no $\Delta$ s.t.:
THM. No universal broadcasting i.e. no $\Delta$ s.t.:
\[
\left( \exists \psi, \phi : \quad \begin{array}{c}
\phi \\
\psi
\end{array} \right) = \left( \begin{array}{c}
\phi \\
\psi
\end{array} \right) \quad \Leftrightarrow \quad \left( \exists \psi', \phi' : \quad \begin{array}{c}
\phi' \\
\psi'
\end{array} \right) = \left( \begin{array}{c}
\phi' \\
\psi'
\end{array} \right)
\]
Prop. Pure reduced state:

\[ \rho \implies \hat{\phi} \]

implies separation:

\[ \rho = \rho' + \hat{\phi} \]
Pf. Since:
**Pf.** Since:

\[
\rho = \begin{array}{c}
\psi \\
\psi
\end{array}
\]

we have:

\[
\begin{array}{c}
\psi \\
\psi
\end{array} = \begin{array}{c}
\phi \\
\phi
\end{array}
\]
i.e.: 

\[\psi = \begin{array}{c}
\end{array}\]
so:

\[
\psi = \psi_1 + \psi_2
\]
and hence:

\[
\begin{array}{c}
\psi \\
\psi
\end{array}
\]

separates:

\[
\begin{array}{cc}
\psi_1 & \psi_1 \\
\psi_2 & \psi_2
\end{array}
\]
Prop. Pure reduced state:

\[ \rho = \hat{\phi} \]

implies separation:

\[ \rho = \rho' \hat{\phi} \]
Prop. Pure reduced process:

\[
\Phi = \hat{f}
\]

implies separation:

\[
\Phi = \rho \hat{f}
\]
Pf. Reduces to previous:
THM. No universal broadcasting i.e. no $\Delta$ s.t.:
Pf. By (l) and Prop:
\textbf{Pf.} By (l) and Prop:

\[
\begin{array}{c}
\Delta \\
\end{array}
\quad = 
\begin{array}{c}
\rho \\
\end{array}
\]

so:

\[
\begin{array}{c}
\rho \\
\end{array}
\quad (r) 
\quad = 
\begin{array}{c}
\Delta \\
\end{array}
\quad = 
\begin{array}{c}
\rho \\
\end{array}
\]

RIP