The Structure of Quantum Computation from Physical Principles

John H. Selby & Ciarán M. Lee

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"The structure of quantum computation..."

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- Quantum departures from classicality
  - Non-locality
  - Contextuality
  - Computational speed-up

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- What provides the quantum speed-up?
- Why is there not more of a speed-up?
- How can we design optimal algorithms?

"...from physical principles."

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Internal perspective:

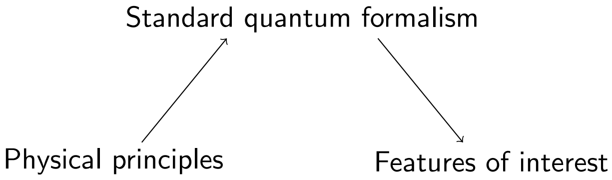
Standard quantum formalism

↓

Features of interest

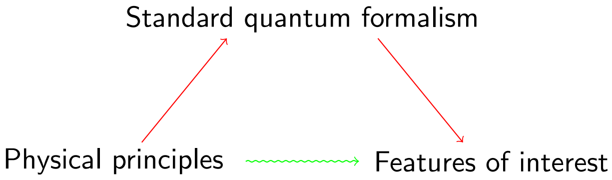
"...from physical principles."

Reconstruction perspective:



"...from physical principles."

External perspective:



Analogy: Study of non-local correlations

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- Information causality and Tsirelson's bound
- Device independent key distribution

Outline

Physical principles

Components of computation

Grover's algorithm

Physical principles

Physical principles

- Causality

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- Purification and purity preservation

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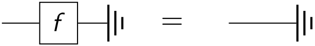
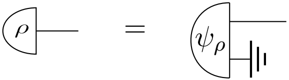
- Causality
- Purification and purity preservation
- Strong symmetry and pure and perfectly distinguishable states

Physical principle 1: Causality



Physical principle 2: Purification and purity preservation



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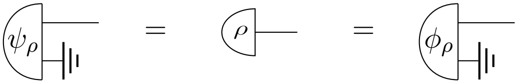

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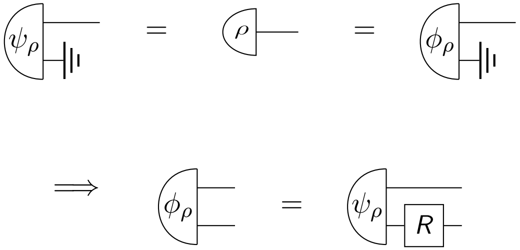

Physical principle 3: Strong symmetry and the existence of pure and perfectly distinguishable states

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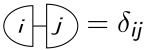
such that



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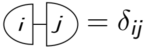

Where

Physical principle 3: Strong symmetry and the existence of pure and perfectly distinguishable states

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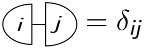
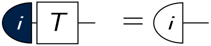
such that



Where

are related by



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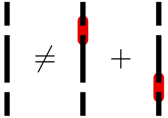

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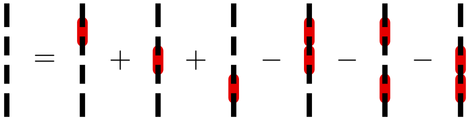

Component 1: Reversible controlled transformations

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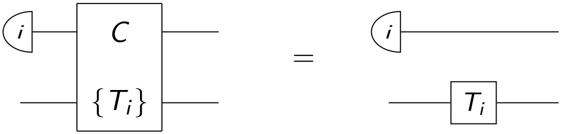

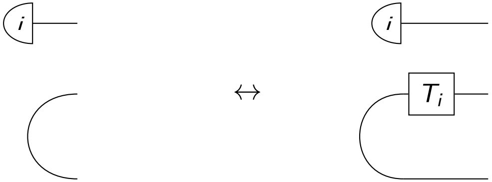
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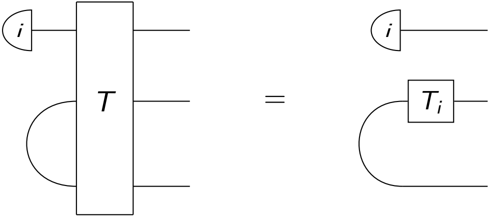
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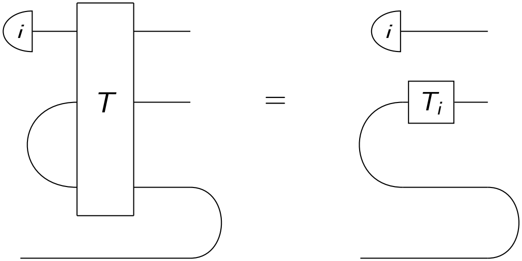
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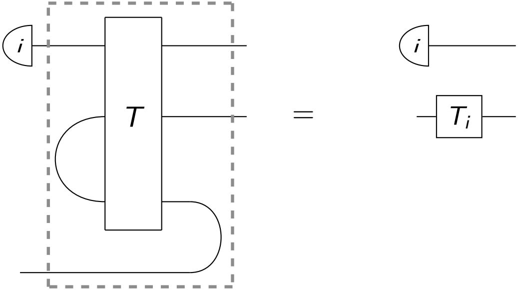
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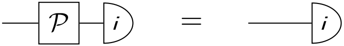
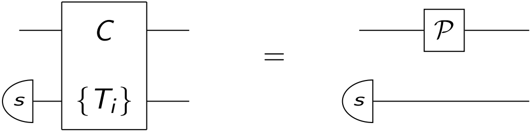
Component 2: Reversible phase transformations



Component 3: Phase kick-back algorithm



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- Oracle problem

The search problem

"Given an N element unstructured list with a unknown marked item x. Then given an oracle $\mathcal{O}_x$ how many queries of $\mathcal{O}_x$ are needed to find x with high probability?"

Quantum oracles

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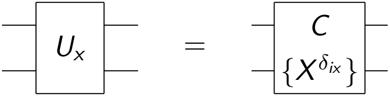

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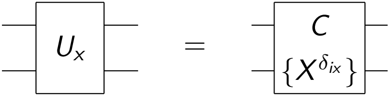

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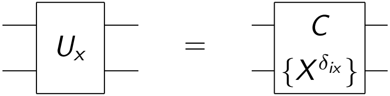

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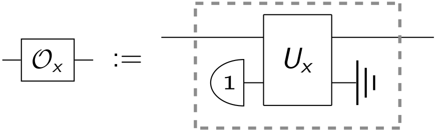

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Controlled transformation

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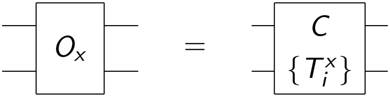
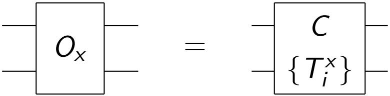


General oracles

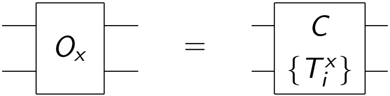
Controlled transformation



Phase transformation

General oracles

Controlled transformation



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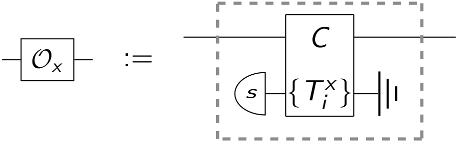

The lower bound

- Classical computers: $O(N)$ queries

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- Quantum computers: $O(\sqrt{N})$ queries

The quantum speed up

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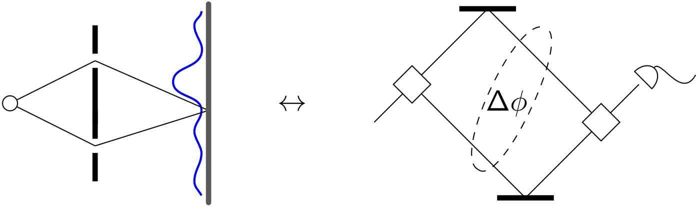

Our result

- Classical computers: $O(N)$ queries
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- Classical computers: $O(N)$ queries
- Quantum computers: $O(\sqrt{N})$ queries
- Computers satisfying our principles: $\Omega(\sqrt{N/h})$ queries

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- Do we need all of these principles?
- Can we reach this lower bound?
- Practical applications?