The Structure of Quantum Computation from Physical Principles

John H. Selby & Ciarán M. Lee

arXiv: 1510.04699, 1604.03118
“The structure of quantum computation...”
“The structure of quantum computation...”

- Quantum departures from classicality
  - Non-locality
  - Contextuality
  - Computational speed-up
“The structure of quantum computation...”

- Quantum departures from classicality
  - Non-locality
  - Contextuality
  - Computational speed-up
“The structure of quantum computation...”

- Quantum departures from classicality
  - Non-locality
  - Contextuality
  - Computational speed-up
- What provides the quantum speed-up?
“The structure of quantum computation…”

- Quantum departures from classicality
  - Non-locality
  - Contextuality
  - Computational speed-up
- What provides the quantum speed-up?
- Why is there not more of a speed-up?
“The structure of quantum computation...”

- Quantum departures from classicality
  - Non-locality
  - Contextuality
  - Computational speed-up
- What provides the quantum speed-up?
- Why is there not more of a speed-up?
- How can we design optimal algorithms?
“...from physical principles.”
“...from physical principles.”

Internal perspective:

Standard quantum formalism

Features of interest
“...from physical principles.”
Reconstruction perspective:

- Standard quantum formalism
  - Physical principles
  - Features of interest
“...from physical principles.”

External perspective:

- Standard quantum formalism
- Physical principles
- Features of interest
Analogy: Study of non-local correlations

- Why are quantum correlations non-local?
Analogy: Study of non-local correlations

- Why are quantum correlations non-local?
- Why are quantum correlations not more non-local?
Analogy: Study of non-local correlations

- Why are quantum correlations non-local?
- Why are quantum correlations not more non-local?
- Information causality and Tsirelson’s bound
Analogy: Study of non-local correlations

- Why are quantum correlations non-local?
- Why are quantum correlations not more non-local?
- Information causality and Tsirelson’s bound
- Device independent key distribution
Outline

Physical principles

Components of computation

Grover’s algorithm
Physical principles
Physical principles

- Causality
Physical principles

- Causality
- Purification and purity preservation
Physical principles

- Causality
- Purification and purity preservation
- Strong symmetry and pure and perfectly distinguishable states
Physical principle 1: Causality
Physical principle 2: Purification and purity preservation
Physical principle 2: Purification and purity preservation

$\psi_\rho \quad = \quad \rho \quad = \quad \phi_\rho$
Physical principle 2: Purification and purity preservation

\[ \psi_\rho = \rho = \phi_\rho \]

\[ \Rightarrow \phi_\rho = \psi_\rho R \]
Physical principle 3: Strong symmetry and the existence of pure and perfectly distinguishable states

There exists $\{i\}_i=0^n$ and $\{j\}_j=0^n$ such that $i_j = \delta_{ij}$.
Physical principle 3: Strong symmetry and the existence of pure and perfectly distinguishable states

There exists

\[ \{ i \}^n_{i=0} \quad \text{and} \quad \{ j \}^n_{j=0} \]

such that

\[ i \}^n_{i=0} = \delta_{ij} \]
Physical principle 3: Strong symmetry and the existence of pure and perfectly distinguishable states

There exists

\[
\begin{align*}
\{i\}^n_{i=0} & \quad \text{and} \quad \{-j\}^n_{j=0}
\end{align*}
\]

such that

\[
\begin{align*}
(i-j) = \delta_{ij}
\end{align*}
\]

Where

\[
\begin{align*}
\{i\}^n_{i=0} & \quad \text{and} \quad \{i\}^n_{i=0}
\end{align*}
\]
Physical principle 3: Strong symmetry and the existence of pure and perfectly distinguishable states

There exists

\[ \{i\}^n_{i=0} \quad \text{and} \quad \{j\}^n_{j=0} \]

such that

\[ i \quad j = \delta_{ij} \]

Where

\[ \{i\}^n_{i=0} \quad \text{and} \quad \{i\}^n_{i=0} \]

are related by

\[ i \quad T = i \]
Do they restrict us to quantum theory?
Do they restrict us to quantum theory?

- Real and fermionic quantum theory
Do they restrict us to quantum theory?

- Real and fermionic quantum theory
- Higher order interference
Do they restrict us to quantum theory?

- Real and fermionic quantum theory
- Higher order interference
Do they restrict us to quantum theory?

- Real and fermionic quantum theory
- Higher order interference
Component 1: Reversible controlled transformations
Component 1: Reversible controlled transformations

\[ C \{ T_i \} = T_i \]
\[ T = T_i \]
Component 2: Reversible phase transformations
Component 3: Phase kick-back algorithm

\[ C \{ T_i \} = P \]
Grover’s algorithm
Grover’s algorithm

- Important quantum algorithm
Grover’s algorithm

- Important quantum algorithm
- Provable advantage
Grover’s algorithm

- Important quantum algorithm
- Provable advantage
- Provably optimal
Grover’s algorithm

- Important quantum algorithm
- Provable advantage
- Provably optimal
- Oracle problem
The search problem

“Given an N element unstructured list with a unknown marked item x. Then given an oracle $O_x$ how many queries of $O_x$ are needed to find x with high probability?”
Quantum oracles
Quantum oracles

\[ U_x |i\rangle |j\rangle = |i\rangle |j \oplus \delta_{ix}\rangle \]
Quantum oracles

\[ U_x |i\rangle |j\rangle = |i\rangle |j \oplus \delta_{ix}\rangle \]

\[ U_x = C\{X^{\delta_{ix}}\} \]
Quantum oracles

\[ U_x |i\rangle |j\rangle = |i\rangle |j \oplus \delta_{ix}\rangle \]

\[ O_x |i\rangle = (-1)^{\delta_{xi}} |i\rangle \]
Quantum oracles

\[ U_x \left| i \right\rangle \left| j \right\rangle = \left| i \right\rangle \left| j \oplus \delta_{ix} \right\rangle \]

\[ O_x \left| i \right\rangle = (-1)^{\delta_{xi}} \left| i \right\rangle \]

\[ C \{ X^\delta_{ix} \} = U_x \]

\[ \text{Diagram} \]
General oracles
General oracles

Controlled transformation
General oracles

Controlled transformation

\[ O_x = C \{ T_i^x \} \]
General oracles

Controlled transformation

\[ O_x = C \{ T_i^x \} \]

Phase transformation
General oracles

Controlled transformation

Phase transformation

\[ O_x = C \{T_i^x\} \]

\[ O_x := C \{T_i^x\} \]
The lower bound

- Classical computers: $O(N)$ queries
The lower bound

- Classical computers: $O(N)$ queries
- Quantum computers: $O(\sqrt{N})$ queries
The quantum speed up
The quantum speed up

- Quantum interference provides the speed up?
The quantum speed up

- Quantum interference provides the speed up?
- More interference gives more of a speed up, e.g. $O(N^{\frac{1}{\hbar}})$?
The quantum speed up

- Quantum interference provides the speed up?
- More interference gives more of a speed up, e.g. $O(N^{\frac{1}{\hbar}})$?
- Interference and phases
The quantum speed up

- Quantum interference provides the speed up?
- More interference gives more of a speed up, e.g. $O(N^{1/\hbar})$?
- Interference and phases
Our result

- Classical computers: $O(N)$ queries
- Quantum computers: $O(\sqrt{N})$ queries
Our result

- Classical computers: $O(N)$ queries
- Quantum computers: $O(\sqrt{N})$ queries
- Computers satisfying our principles: $\Omega(\sqrt{N/h})$ queries
Conclusion
Conclusion

- Physical principles
Conclusion

- Physical principles
  - elementary components of computation
Conclusion

- Physical principles
  - elementary components of computation
  - the quantum lower bound
Conclusion

- Physical principles
  - elementary components of computation
  - the quantum lower bound
- Do we need all of these principles?
Conclusion

- Physical principles
  - elementary components of computation
  - the quantum lower bound
- Do we need all of these principles?
- Can we reach this lower bound?
Conclusion

- Physical principles
  - elementary components of computation
  - the quantum lower bound
- Do we need all of these principles?
- Can we reach this lower bound?
- Practical applications?