Tight reference frame-independent quantum teleportation

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- No transfer of information about the RF configuration.
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- Depends on the action of the group of RF transformations.
  The group must be finite.
- Constructions of reference frame–independent (RFI) teleportation protocols.

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- CQM as a way to work with RFs in quantum information.

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- When RF configuration transforms by $g^{-1} \in G$:

	Old frame	New frame
State	$\|\psi\rangle$	$\pi(g)\,\|\psi\rangle$
Operations	$L \in \text{End}(H)$	$\pi(g) L \pi(g)^\dagger$

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  1. U_i are all unitary.
  2. U_i are orthonormal under the Hilbert-Schmidt inner product.
- When Hilbert spaces have minimal dimension, teleportation protocols correspond to UEBs:

Shared entangled state	$\sum_i \|i\rangle \otimes \|i\rangle$
Alice's measurement basis	$\|\phi_x\rangle := \sum_i \|i\rangle \otimes U_x \|i\rangle$
Bob's unitary correction	$C_x := U_x^T$

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Example: II

- Take $G = \mathbb{Z}^2$, where the nontrivial element $a \in G$ acts as

$$\pi(a) = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ 1/2 & -\sqrt{3}/2 \end{pmatrix}.$$

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$$\pi(a) = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ 1/2 & -\sqrt{3}/2 \end{pmatrix}.$$

- They agree to use the following UEB:

$$U_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \qquad U_2 = \frac{1}{4} \begin{pmatrix} -\sqrt{2} - \sqrt{6} & -\sqrt{2} + \sqrt{6} \\ -\sqrt{2} + \sqrt{6} & \sqrt{2} + \sqrt{6} \end{pmatrix}$$

$$U_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \qquad U_3 = \frac{1}{4} \begin{pmatrix} \sqrt{2} - \sqrt{6} & -\sqrt{2} - \sqrt{6} \\ -\sqrt{2} - \sqrt{6} & -\sqrt{2} + \sqrt{6} \end{pmatrix}$$

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  - Alice's perspective: Bob's correction was *not* U_i^T but rather $\pi(a)^\dagger U_i^T \pi(a)$.
  - Bob's perspective: the measurement result i Alice communicated did not correspond to the state $(\mathbb{1} \otimes \pi(a)) |\phi_i\rangle$ she measured.

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If Alice sends her measurement result to Bob as speakable information, the procedure only succeeds for all RF alignments when G acts by a global phase.

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- Therefore all irreducible factors of H are identical and 1D.

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- Suppose Bob's lab is aligned upside-down wrt Alice's. 0, 1, 2 or 3 will be received as 1, 0, 3 or 2 respectively.

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- So teleportation is successful regardless of RF alignment!

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  2. The classical channel carried a G-action, so we could encode the measurement results to carry the inverse permutation to the UEB.
- If we can find a suitable classical channel and a G-equivariant UEB, we can perform RFI teleportation.

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This procedure is RF–independent exactly when the UEB is G-equivariant.

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- Alice can encode her measurement result in the time of arrival of signals.

Finding G-equivariant UEBs

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- We provide a method for showing non-existence.
- We provide methods for constructing them when they do.

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- G-equivariant orthonormal bases can be easily classified!

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  $\iff$ they correspond to conjugate subgroups.
- $\mathcal{M}$ is additive ($\sqcup \rightarrow \oplus$).
- So in order to classify all objects in $Im(\mathcal{M})$, it is sufficient to find the images of the coset spaces G/H under $\mathcal{M}$ - the basic permutation representations.

A representation on which no G-equivariant UEBs exist

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- This does not work for all irreps! (2D irrep of D_8.)

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Theorem (VV)

Let $|v_i\rangle$ be the G-equivariant orthonormal basis, and H be the Hadamard matrix. Then the G-equivariant UEB is:

$$(U_H)_{ij} = \frac{1}{N} H \circ diag(H, j)^{\dagger} \circ H^{\dagger} \circ diag(H^T, i) \qquad (1)$$

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- $\pi(G) < S_d$ so the worst case is $\pi(G) = S_d$.
- Any $M < C(S_d)$ has identical entries on the diagonal and identical entries everywhere else.

A sufficient condition for dim < 5 (cont.)

Proof (Cont.)

Unitarity of such a matrix is equivalent to

$$|b|^2 = \frac{1 - |a|^2}{d - 1} \tag{2}$$

$$\Re(a^* b) = \frac{2 - d}{2} |b|^2. \tag{3}$$

These equations can be satisfied for $|a|, |b| = \frac{1}{\sqrt{d}}$ iff $d < 5$. □

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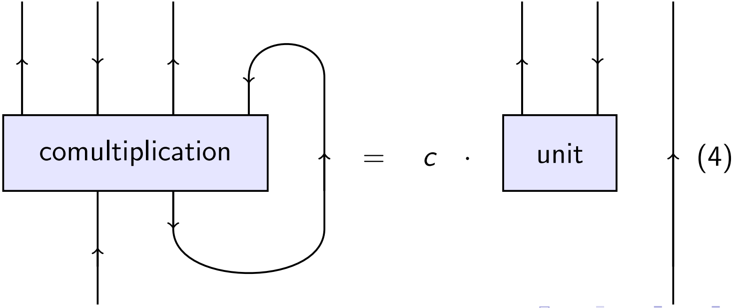
- In a dagger-compact category, a *quantum teleportation procedure* is a classical structure on $A \otimes A^*$ satisfying:



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- All categorical constructions of UEBs carry over to **Rep(**G**)**.
- Unclear how to generalise non-categorical constructions to the G-equivariant setting.

Future work

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