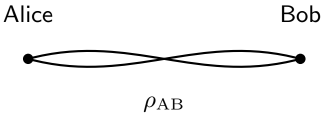
Postquantum steering

Ana Belén Sainz, Nicolas Brunner, Daniel Cavalcanti
Paul Skrzypczyk and Tamás Vértesi

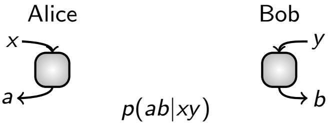
Phys. Rev. Lett. 115, 190403 (2015)

Entanglement



- Quantum teleportation
- Quantum Key Distribution

Nonlocality



- Device Independent QKD
- Randomness

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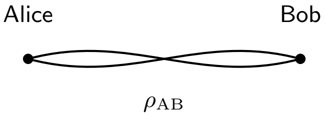

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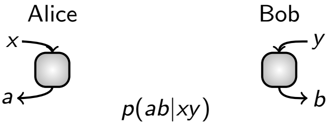

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Steering

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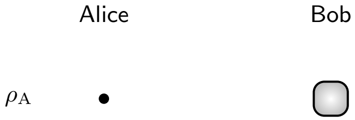

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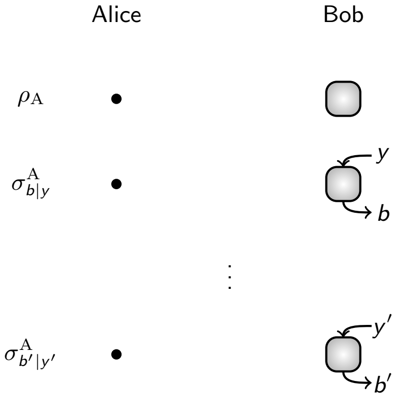

Alice
Bob
ρ_{A}
$\sigma^{\mathrm{A}}_{b|y}$
y
b
$\vdots$
$\sigma^{\mathrm{A}}_{b'|y'}$
y'
b'

Steering

Fix y $\longrightarrow$ ensemble: $\{\sigma^{\mathrm{A}}_{b|y}\}_b\,, \quad p(b|y) = \mathrm{tr}\left(\sigma^{\mathrm{A}}_{b|y}\right)\,, \quad \rho_{\mathrm{A}} = \sum_b \sigma^{\mathrm{A}}_{b|y}$

Assemblage: $\{\sigma^{\mathrm{A}}_{b|y}\}_{b,y}$.

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Quantum: $\sigma^{\mathrm{A}}_{b|y} = \mathrm{tr}_{\mathrm{B}}\left(\mathbb{1}_{\mathrm{A}} \otimes M_{b|y}\,\rho_{\mathrm{AB}}\right)$

Given an assemblage, **could it have a classical explanation?**

Here:

Given an assemblage, **could it have a quantum explanation?**

Bipartite steering

$$\text{Given } \{\sigma^{\mathrm{A}}_{b|y}\}_{b,y}, \quad \rho_{\mathrm{A}} = \textstyle\sum_b \sigma^{\mathrm{A}}_{b|y}, \quad \mathrm{tr}\left(\rho_{\mathrm{A}}\right) = 1$$

$$\exists\, \rho_{\mathrm{AB}}, \quad \{M_{b|y}\}_{b,y} \quad \text{st} \quad \sigma^{\mathrm{A}}_{b|y} = \mathrm{tr}_{\mathrm{B}}\left(\mathbb{1}_{\mathrm{A}} \otimes M_{b|y}\, \rho_{\mathrm{AB}}\right)$$

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- Alice and Bob: **Yes !** GHJW theorem[1]
- **Multipartite scenarios?**

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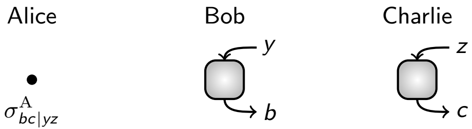
Steering: multipartite scenarios



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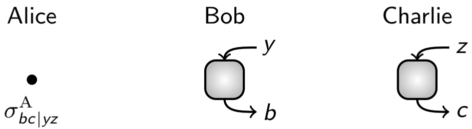

Fix y, z, ensemble: $\{\sigma^{\mathrm{A}}_{bc|yz}\}_{b,c}$, $\quad p(bc|yz) = \mathrm{tr}\left(\sigma^{\mathrm{A}}_{bc|yz}\right)$, $\quad \rho_{\mathrm{A}} = \sum_{b,c} \sigma^{\mathrm{A}}_{bc|yz}$

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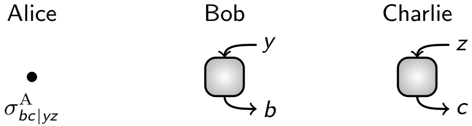

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No Signalling: $\sum_b \sigma^{\mathrm{A}}_{bc|yz} = \sum_b \sigma^{\mathrm{A}}_{bc|y'z}$, $\quad \sum_c \sigma^{\mathrm{A}}_{bc|yz} = \sum_c \sigma^{\mathrm{A}}_{bc|yz'}$

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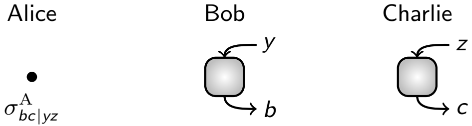

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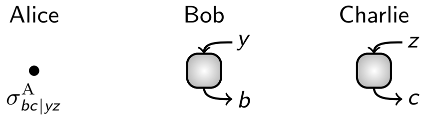
$\exists\, \rho_{\mathrm{ABC}}$, $\{M_{b|y}\}_{b,y}$, $\{M_{c|z}\}_{c,z}$ **st** $\sigma^{\mathrm{A}}_{bc|yz} = \mathrm{tr}_{\mathrm{B}}\left(\mathbb{1}_{\mathrm{A}} \otimes M_{b|y} \otimes M_{c|z}\, \rho_{\mathrm{ABC}}\right)$

Postquantum steering: example



$$b, c, y, z \in \{0, 1\}$$

$$\rho_{\mathrm{A}} = \frac{\mathbb{1}}{2}\,.$$

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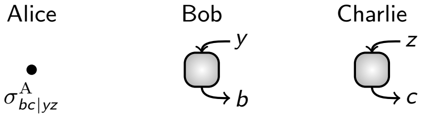

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- $(y, z) = (0, 0)\,, (0, 1)\,, (1, 0)$:

$$\sigma^{\mathrm{A}}_{bc|yz} = \begin{cases} \frac{\mathbb{1}}{4}\,, & \text{if } b = c\,, \\ 0\,, & \text{if } b \neq c\,, \end{cases}$$

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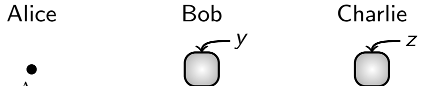

$\sigma^{\mathrm{A}}_{bc|yz}$

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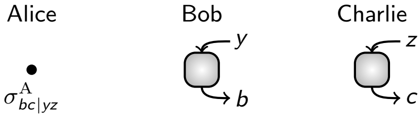
$$p(bc|yz) = \mathrm{tr}\left(\sigma^{\mathrm{A}}_{bc|yz}\right) \overset{quantum}{=} \mathrm{tr}_{\mathrm{BC}}\left(M_{b|y} \otimes M_{c|z}\,\rho_{\mathrm{BC}}\right)$$

Postquantum steering: example



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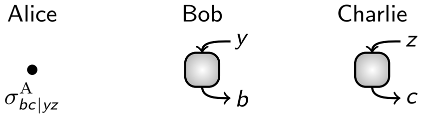
$$p(bc|yz) = \begin{cases} \frac{1}{2}\,, & \text{if } b \oplus c = yz\,, \\ 0\,, & \text{otherwise}\,. \end{cases}$$

No quantum realisation for the assemblage

Postquantum steering without postquantum NL



(1) Postquantum assemblage $\left\{\sigma^{\mathrm{A}}_{bc|yz}\right\}_{b,y,c,z}$

(2) Quantum correlations for every measurement by Alice:

$$p(abc|xyz) = \mathrm{tr}\left(M_{a|x}\,\sigma^{\mathrm{A}}_{bc|yz}\right)$$

(1) Postquantum assemblage $\sigma^{\mathrm{A}}_{bc|yz}$

Steering inequality: F_{bcyz}

$$S(\{\sigma^{\mathrm{A}}_{bc|yz}\}) := \mathrm{tr} \sum_{bcyz} F_{bcyz}\, \sigma^{\mathrm{A}}_{bc|yz}$$

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How to compute β_{Q}? $\rightarrow$ upper bound

Almost quantum assemblages: $\widetilde{\mathrm{Q}} \supset \mathrm{Q}$

$$\max_{\{\sigma^{\mathrm{A}}_{bc|yz}\} \in \widetilde{\mathrm{Q}}} S(\{\sigma^{\mathrm{A}}_{bc|yz}\}) =: \beta_{\widetilde{\mathrm{Q}}} \ge \beta_{\mathrm{Q}}$$

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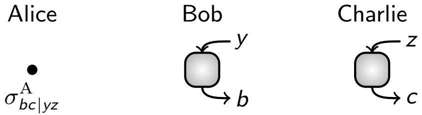
$S(\{\sigma^{\mathrm{A}}_{bc|yz}\}) > \beta_{\widetilde{\mathrm{Q}}} \quad \Rightarrow \quad \sigma^{\mathrm{A}}_{bc|yz}$ **is postquantum**

Example without postquantum nonlocality



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(2) Quantum correlations $p(abc|xyz)$

(i) $p(abc|xyz)$ is local

(ii) Qubit assemblage (real): local for all projective measurements by Alice

(iii) Qutrit assemblage, local for all POVMs[2].

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$\sigma^{\mathrm{A}}_{bc|yz}$ local for $\{x_1, \ldots, x_m\}$ $\quad\Leftrightarrow\quad$ $\sigma^{\mathrm{A}}_{bc|yz}(\mu)$ local $\forall\, \Pi_{a|x}$

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Example without postquantum nonlocality

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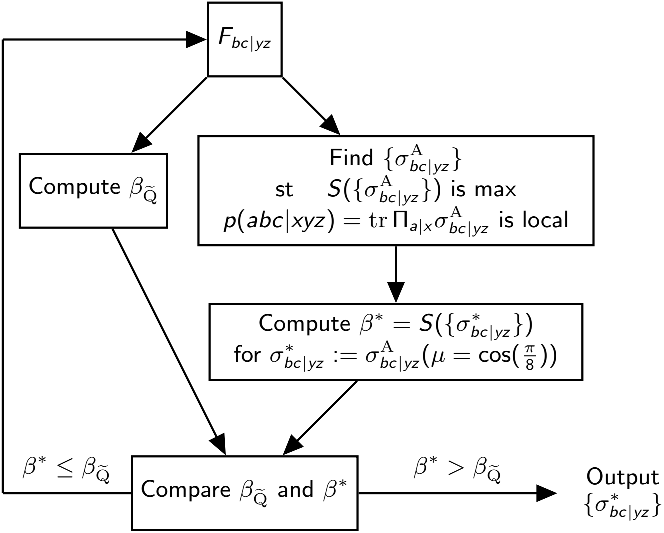
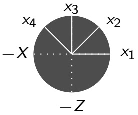
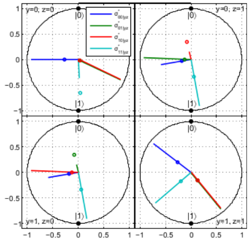


Fix dichotomic measurements $\Pi_{a|x}$:



Example without postquantum nonlocality

(ii) $\{\sigma^*_{bc|yz}\}$: $\begin{cases} \text{- it is postquantum,} \\ \text{- } p(abc|xyz) \text{ is local for every projective measurement by Alice.} \end{cases}$



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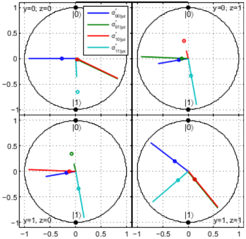

(iii) $\tilde{\sigma}^*_{bc|yz} = \frac{1}{3}\,\sigma^*_{bc|yz} + \frac{2}{3}\,\mathrm{tr}\left(\sigma^*_{bc|yz}\right)|2\rangle\langle 2|$

$\{\tilde{\sigma}^*_{bc|yz}\}$ is a postquantum qutrit assemblage

that always gives quantum correlations for POVMs

Summary and open questions

- Steering beyond quantum theory $\rightarrow$ multipartite scenarios
- Genuinely new effect
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- Insight on the characterisation of quantum phenomena
- General framework for non-signalling assemblages
  $\rightarrow$ quantify postquantumness
- Information-theoretic applications of postquantum steering

Thanks !!!

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(ii) Qubit assemblage, local for all PVM

$$\Pi_{a|x}(\mu) = \mu\, \Pi_{a|x} + (1-\mu)\, \mathbb{1}/2, \quad \sigma^{\mathrm{A}}_{bc|yz}(\mu) = \mu\, \sigma^{\mathrm{A}}_{bc|yz} + (1-\mu)\, \mathrm{tr}\left(\sigma^{\mathrm{A}}_{bc|yz}\right) \mathbb{1}/2$$

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- $p(abc|xyz) = \mathrm{tr}_{\mathrm{A}} \left(\Pi_{a|x}(\mu)\, \sigma^{\mathrm{A}}_{bc|yz}\right) = \mathrm{tr}_{\mathrm{A}} \left(\Pi_{a|x}\, \sigma^{\mathrm{A}}_{bc|yz}(\mu)\right)$
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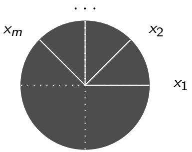

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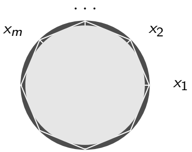

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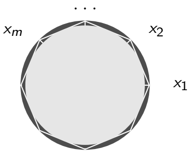

$$\sigma^{\mathrm{A}}_{bc|yz} \textbf{ local for } \{x_1, \ldots, x_m\} \quad \Leftrightarrow \quad \sigma^{\mathrm{A}}_{bc|yz} \textbf{ local for } \Pi_{a|x}(\mu)$$

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