Quantum Foundations and Bell's Theorem

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Outline

- Characteristics of Quantum Theory
- Local Realism and Bell's Theorem
- Contextuality
- The Reality of the Wavefunction

Disclaimer

- I’m not in Foundations, Outsider’s impression
- Few technical details, except where simple
- Mainly to give a flavour of the issues in Quantum Foundations
- Highly incomplete (possible wrong in parts)

(Some) Questions in Quantum Foundations

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- Meaning of the wavefunction?
- Meaning of measurement?
- One world or many?
- Real or not?
- Local or not?
- Difference between Classical and Quantum?
- Why is QM the way it is, not some other theory?

Quantum Theory in a Nutshell

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- (Pure) state of a system represented by a vector in a complex Hilbert Space
- Observables represented by Hermitian operators
- Probabilistic outcomes of measurements
- State modified by measurement
- Heisenberg’s uncertainty leads to impossibility of simultaneous definite values for all properties
- Entanglement, non-locality

How Quantum is Different from Classical

- Classical theories
  - Allows definite (macro realistic) states of systems
  - Measurement just reveals state, noiseless in principle
- Quantum theory
  - Allows superposition of states
  - Distinct states may not be different (non-orthogonality)
  - Measurement intrinsically disturbing

Three strands to Foundations

- Looking for novel effects in quantum theory;
- Investigating conceptual issues in, and interpretations of, quantum theory; and
- Developing a deeper understanding of the structure of the theory (both mathematical and conceptual) for its own sake, for the purposes of finding a way to reconstruct the theory from more basic axioms, and for the purpose of going beyond quantum theory.

https://www.perimeterinstitute.ca/research/research-areas/quantum-foundations/more-quantum-foundations

The Danger Zone: Interpretations

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- Copenhagen (?)
- Many Worlds/Minds
- Shut up and calculate, non-interpretation
- Epistemic (states of knowledge)
- De Broglie-Bohm (non-local but realist)

We’ll ignore these issues here,
save it for discussion over a pint

Two Main Approaches to Understanding QM

- Accept the classical world view
  – Find a way of interpreting/modifying quantum theory to fit, e.g. hidden variables.
- Accept quantum theory
  – Find a way by which the classical world emerges, e.g. decoherence programme

Einstein-Podolsky-Rosen (EPR 1935)

- Argued QM Incomplete
  - Probabilities of measurement outcomes due to ignorance of the actual underlying physical state
  - Appeared to sidestep Heisenberg's Uncertainty

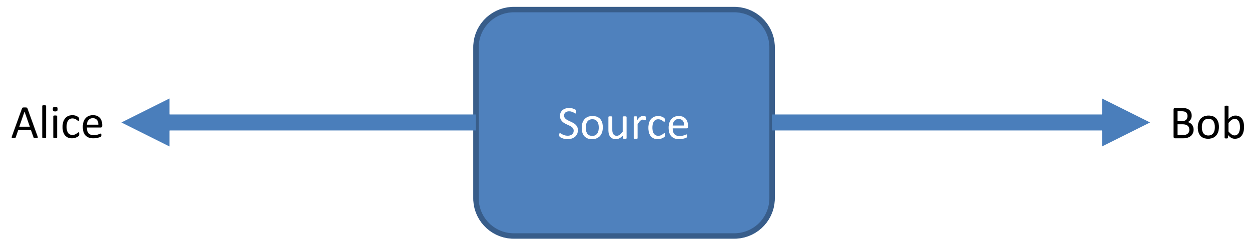
EPR

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

Position x
Momentum p

$$[x, p] = i\hbar$$

QM says a system cannot have simultaneous definite values for both x, p



$$|\Psi\rangle_{AB} = \sum_x |x\rangle_A \otimes |x\rangle_B = \sum_p |p\rangle_A \otimes |-p\rangle_B$$

- If Alice measures x, can predict Bob would have measured x as well, therefore Bob must have had x all along
- Conversely, if Bob measures p, he can predict Alice would have measured –p as well, hence she must have had –p all along
- Hence, they jointly could conclude that they both had particles with definite position and momentum all along, in contradiction with QM

EPR Summary

- EPR assumptions
  - Locality, Alice’s choice of measurement (position or momentum) does not influence the results of Bob’s measurement
  - Counterfactual reasoning, Alice concludes about the results of a measurement by Bob that isn’t performed, vice versa
- EPR Concludes QM Incomplete. The system of two particles are in a definite physical state. A complete physical theory should be able to describe the state in terms of definite outcomes of any possible set of measurements.

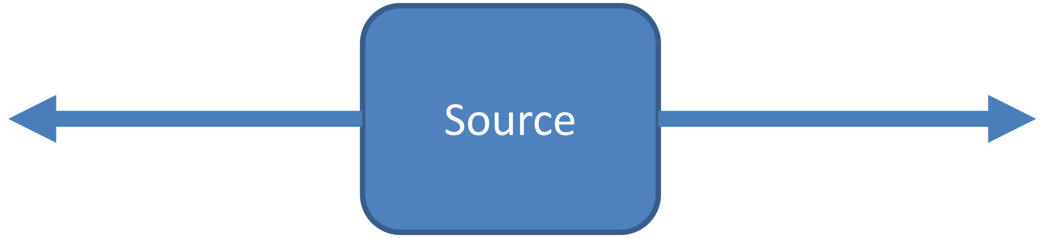
Bell’s Theorem

- How to test the “Classical Assumptions”?
  - Realism, underlying “hidden variables” that determine results of all measurements
  - Locality, the actions at one point cannot instantaneously influence the results at another
- Bell’s Theorem/Inequality
  - Takes the two assumptions above
  - Plus other “reasonable” assumptions
  - Finds an observable limit to such theories having these assumptions
  - QM “violates” this limit

Clauser-Horne-Shimony-Holt (CHSH)

Alice and Bob choose their measurements randomly and independently

Alice can measure property A_1 or A_2



Bob can measure property B_1 or B_2

Four possible sets of joint measurements:
(A_1,B_1), (A_1,B_2), (A_2,B_1), (A_2,B_2)

Each measurement has two possible outcomes, a,b = +1 or -1

Correlation function for (A_j,B_k)

$$\langle A_j B_k \rangle = (+1)P\left(a=+1, b=+1 \mid A_j, B_k\right) + (-1)P\left(a=-1, b=+1 \mid A_j, B_k\right)$$
$$+(-1)P\left(a+1, b=-1 \mid A_j, B_k\right) + (+1)P\left(a=-1, b=-1 \mid A_j, B_k\right)$$

CHSH Inequality

$$\left|\langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle\right| \leq 2$$

Bell's Theorem Example

Alice

Bob

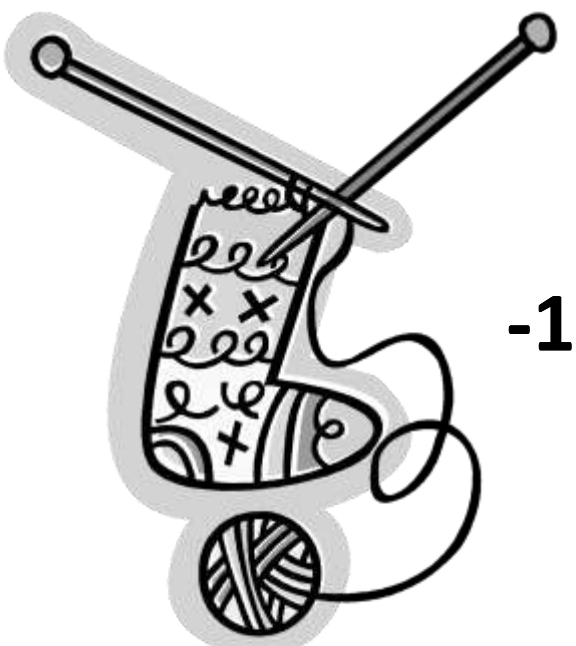
+1

Little

or

Big

+1



-1

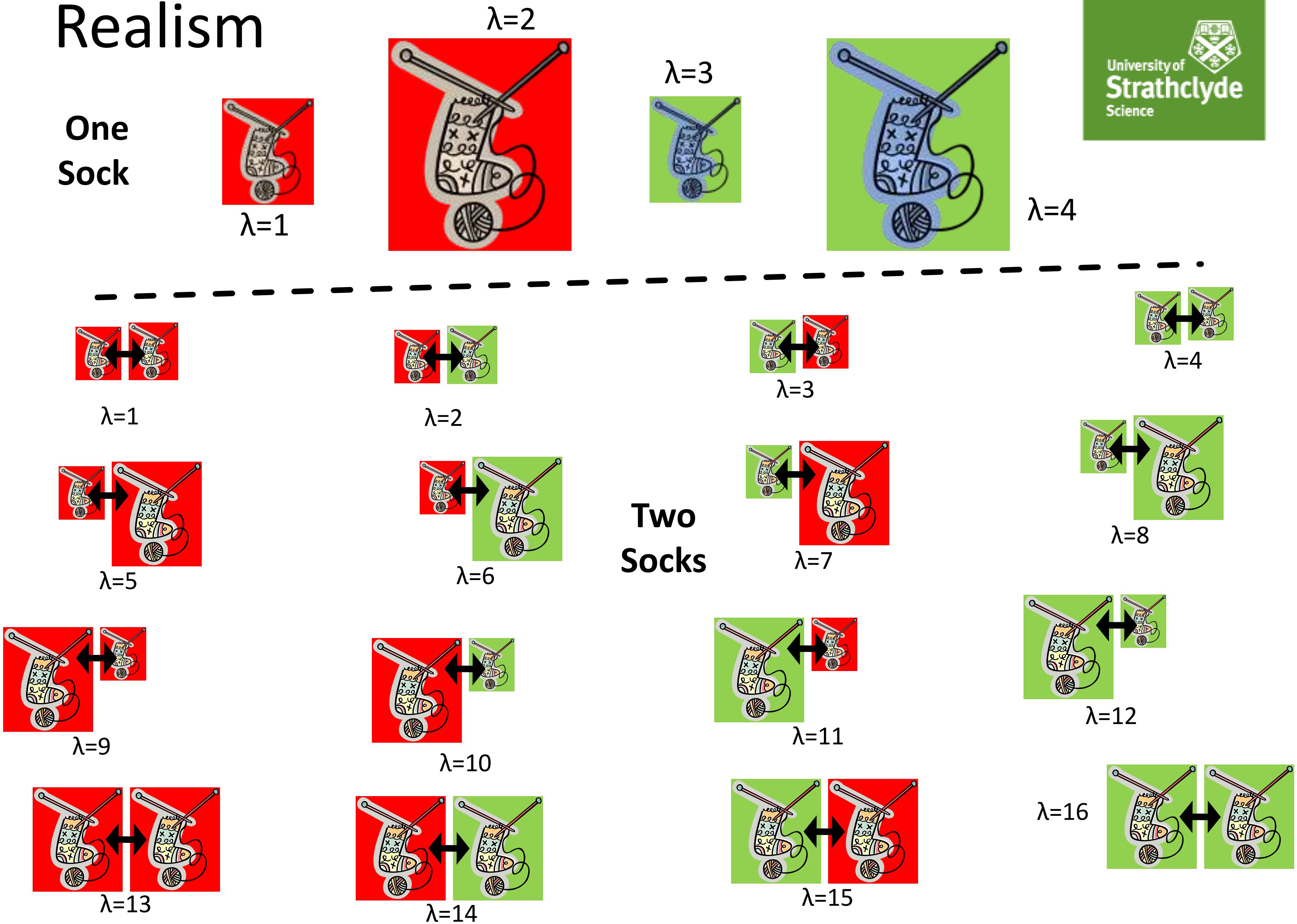
Green

or

Red

-1

Realism



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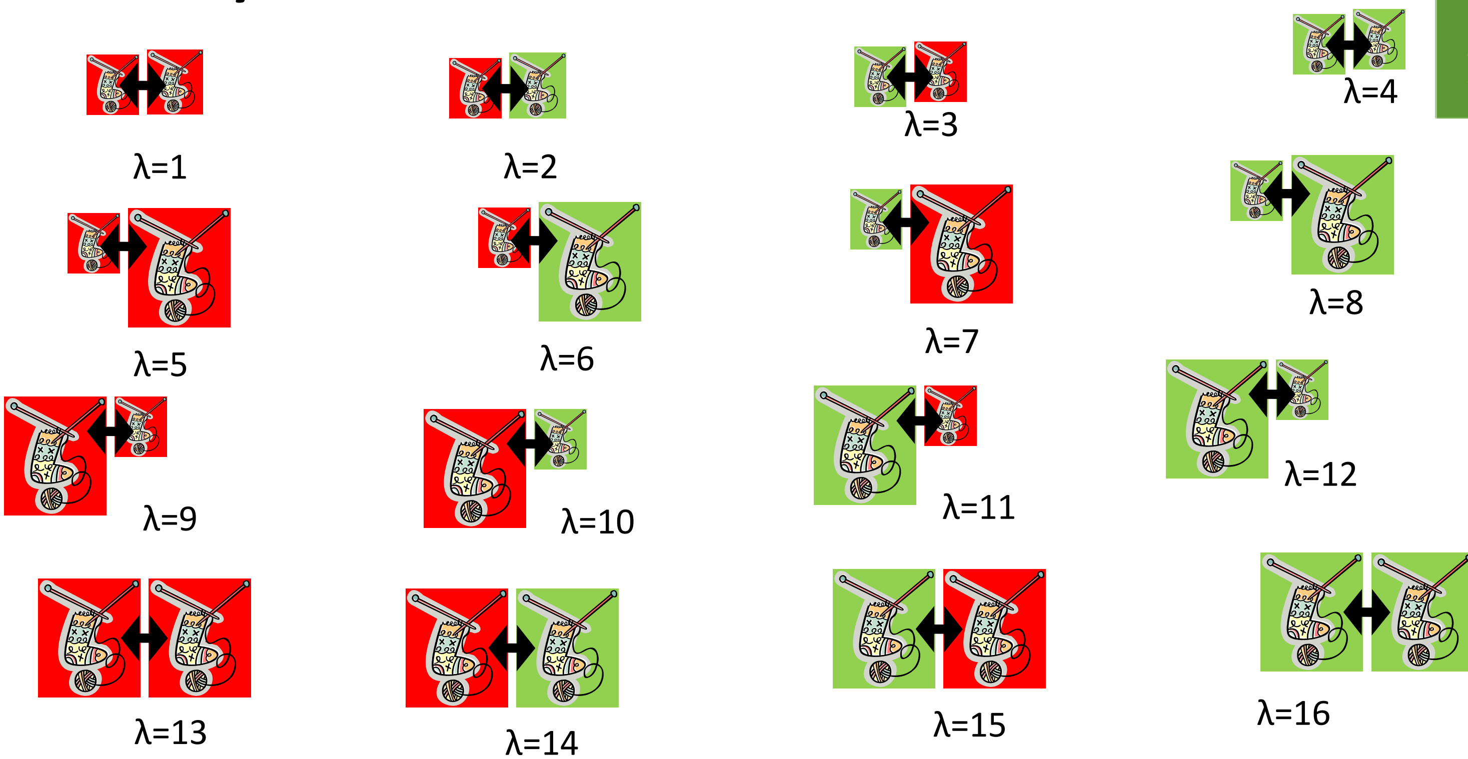
Locality



- The outcome of Alice’s measurement does not depend on the choice of measurement by Bob.

- E.g. Bob’s decision to look at size or colour does not swap Alice’s sock.

- Alice’s sock is only pre-determined by λ.

CSHS Inequality Cont.

$$\left|\langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle\right| = S$$

Fix λ. Assume definite values for A_1, A_2, B_1, B_2 exist simultaneously

$$\begin{aligned}\left|\langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle\right| &= \left|A_1 B_1 + A_1 B_2 + A_2 B_1 - A_2 B_2\right| \\ &= \left|A_1 (B_1 + B_2) + A_2 (B_1 - B_2)\right| \\ &= 2\end{aligned}$$

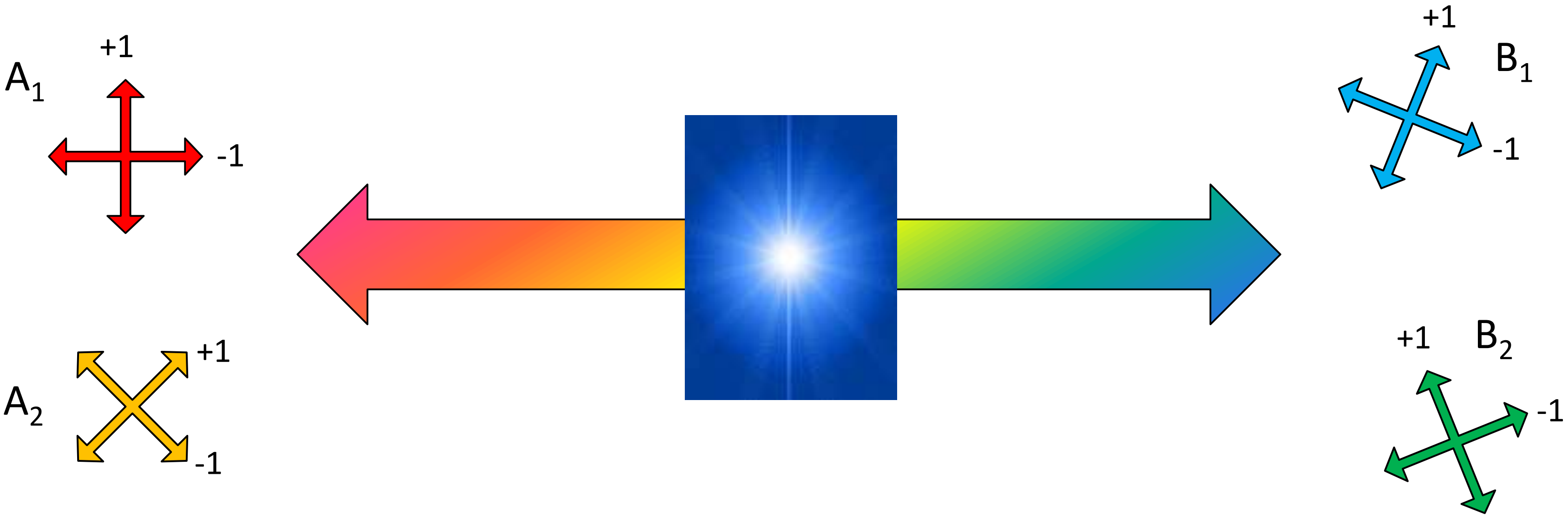
Alice’s choice does not affect Bob’s values

Any mixture of λ cannot increase this value.

For local realistic theories, $S \leq 2$

QM and Local Realism

Example: Two maximally polarisation-entangled photons



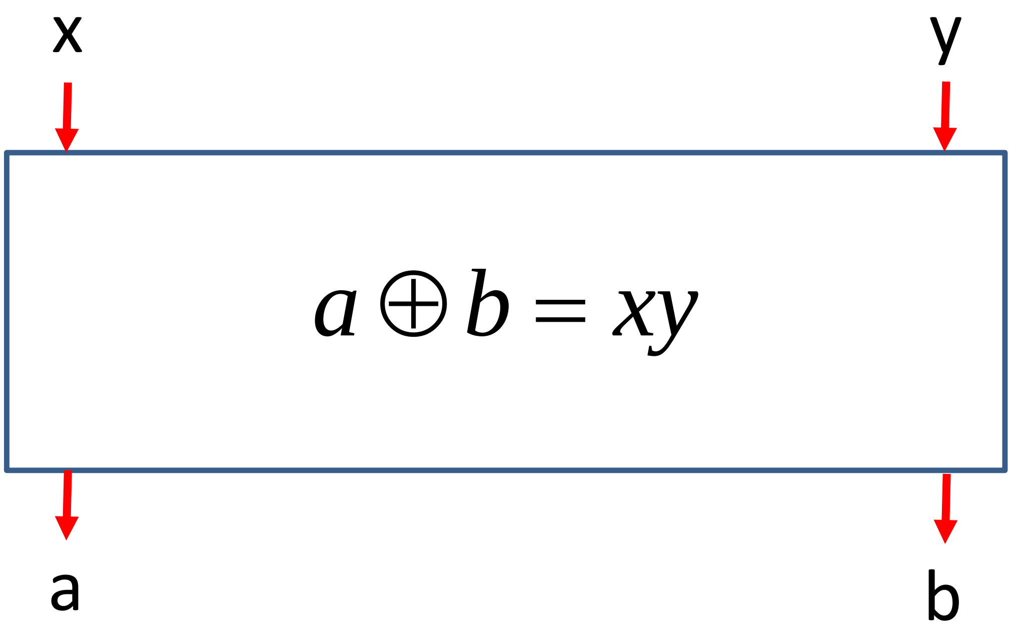
Quantum Mechanics $|S| \leq 2\sqrt{2}$

Note on Loopholes

- Assumptions/loopholes
  - No post-selection, fair-sampling, high detection efficiency
  - Locality, measurements occur faster than light time of flight between Alice and Bob
  - Coincidence loophole
  - Independence of measurement settings
  - Memory loophole
  - Superdeterminism

Popescu-Rohrlich Boxes

x, y, a, b = 0,1



Non-signalling: Alice's result does not say anything about Bob's choice

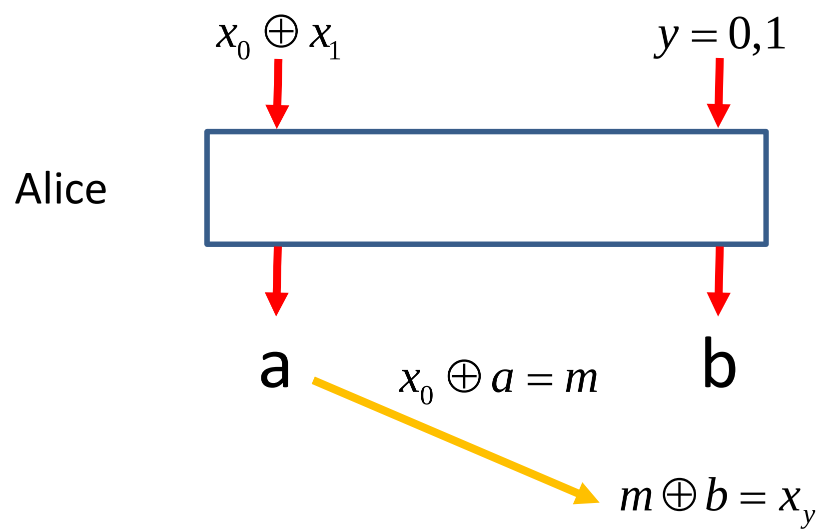
S=4 Stronger non-locality than QM

QM can output required function with $p = \frac{\left(2+\sqrt{2}\right)}{4} \approx 0.85$

Causality and non-locality as axioms for quantum Mechanics, Popescu, S. & Rohrlich, D., Found. Phys. 24, 379–385 (1994).

Information Causality

- Alice wants Bob to have access to 2 bits of information but can only send 1
- With PR Boxes, Bob can independently decide which bit to retrieve



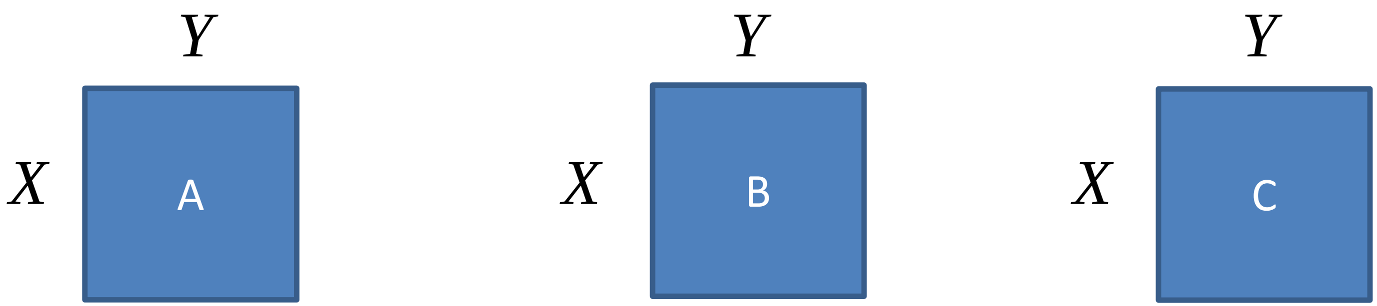
- QRAC $p = \cos^2\left(\pi/8\right) \approx 0.85$
- In QM, m transmitted bits allows access to at most data set size m

Information causality as a physical principle. Pawlowski, M. et al., Nature 461, 1101 (2009).

GHZ(M) "Paradox"

- Three-party, "deterministic" counterexample to local realism

$$|GHZ\rangle_{ABC} = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$



$$\langle X \otimes Y \otimes Y \rangle = -1$$

$$\langle Y \otimes X \otimes Y \rangle = -1$$

$$\langle Y \otimes Y \otimes X \rangle = -1$$

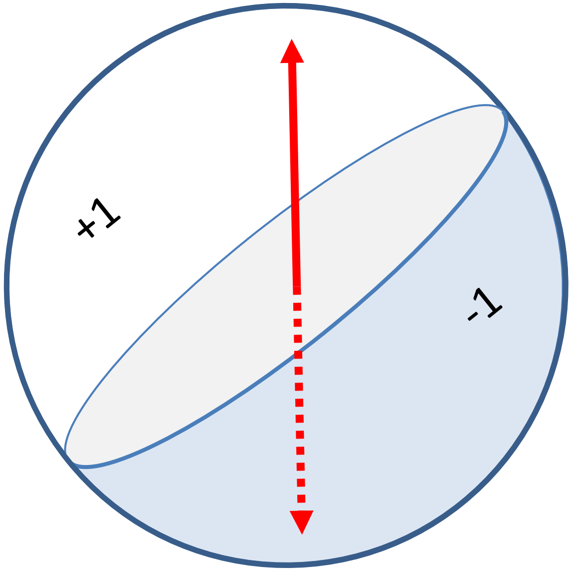
$$\langle X \otimes X \otimes X \rangle = +1$$

Local Realistic Model -1

(Non-)Contextuality

- Non-contextuality
  - All outcomes of measurements represent "elements of reality"
  - All observables defined for a QM system have definite values at all times
  - Underlying physical reality has definite outcomes regardless of configuration of measurements
- The non-commutivity of QM results in contextuality in higher than 3 dimensions

Sketch of Non-contextual assignment of projection outcomes for a qubit



(Bell-)Kochen-Specker Theorem

- In Hilbert space of dimension 3, 117 projections cannot simultaneously be ascribed definite outcomes consistently
- Easier proof in 4 dimensions (Cabello et al 1997)

u_1	(0, 0, 0, 1)	(0, 0, 0, 1)	(1, −1, 1, −1)	(1, −1, 1, −1)	(0, 0, 1, 0)	(1, −1, −1, 1)	(1, 1, −1, 1)	(1, 1, −1, 1)	(1, 1, 1, −1)
u_2	(0, 0, 1, 0)	(0, 1, 0, 0)	(1, −1, −1, 1)	(1, 1, 1, 1)	(0, 1, 0, 0)	(1, 1, 1, 1)	(1, 1, 1, −1)	(−1, 1, 1, 1)	(−1, 1, 1, 1)
u_3	(1, 1, 0, 0)	(1, 0, 1, 0)	(1, 1, 0, 0)	(1, 0, −1, 0)	(1, 0, 0, 1)	(1, 0, 0, −1)	(1, −1, 0, 0)	(1, 0, 1, 0)	(1, 0, 0, 1)
u_4	(1, −1, 0, 0)	(1, 0, −1, 0)	(0, 0, 1, 1)	(0, 1, 0, −1)	(1, 0, 0, −1)	(0, 1, −1, 0)	(0, 0, 1, 1)	(0, 1, 0, −1)	(0, 1, −1, 0)

18 unique vectors

$$P_j = \frac{|u_j\rangle\langle u_j|}{\langle u_j | u_j\rangle} \qquad \mathbf{1} = P_1 + P_2 + P_3 + P_4$$

Impossible to only assign a single 1 and three 0s to each column consistently

Trivial proof, odd versus even

Contextuality and Bell

- Bell Non-Locality a form of Contextuality
- Locality imposes contextual constraint

Reality of the Wavefunction

- Ontic
  - Wavefunction is “real”
  - Wavefunction represents the physical state
- Epistemic
  - Wavefunction is a “state of knowledge”
  - Exists deeper layer of physical reality, wavefunction is a statistical description

Epistemic vs Ontic

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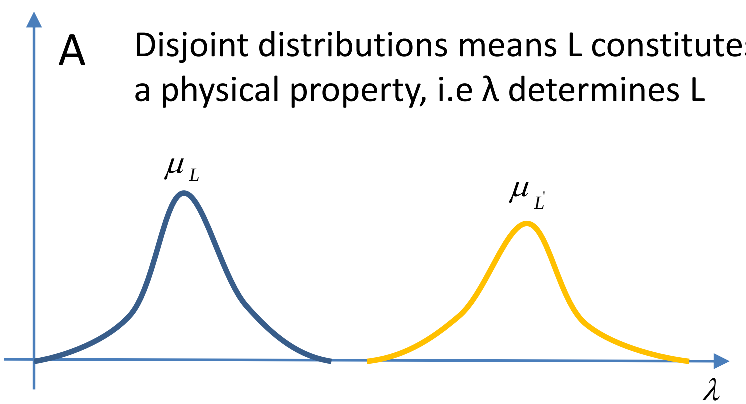
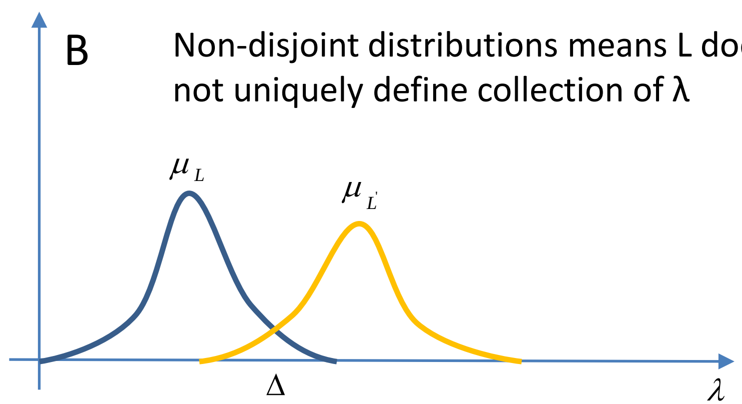
- Is the wavefunction real?
  - Ψ-Epistemic: State of knowledge. The same actual physical state could be part of the ensembles for two different wavefunctions. “Collapse”=Bayesian Update.
  - Ψ-Ontic: Real in the sense that different wavefunctions represent different underlying physical configurations.



Ontic



Epistemic

Epistemic Approaches

- Reproduce "quantum" features from underlying epistemic toy models
  - E.g. Spekkens Toy Model
- Cannot reproduce all quantum phenomena
  - E.g. Bell violations, BKS

Pusey-Barrett-Rudolph (PBR)

- Under some “natural assumptions”, wavefunction cannot be interpreted statistically
  - There exists a real physical state, objective and independent of observer
  - Systems can be prepared independently

$$|\psi_0\rangle = |0\rangle$$

$$|\psi_1\rangle = |+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$\langle \psi_0 | \psi_1 \rangle = \frac{1}{\sqrt{2}}$$

Epistemic view: overlap in actual underlying distribution of states

$\Delta \neq \varnothing$

Independently prepare

$|\psi_j\rangle \otimes |\psi_k\rangle$

Some probability that (λ_1, λ_2) compatible with all four possible states

$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle\right)$$

$$|\xi_2\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \otimes |-\rangle + |1\rangle \otimes |+\rangle\right)$$

$$|\xi_3\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle \otimes |1\rangle + |-\rangle \otimes |0\rangle\right)$$

$$|\xi_4\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle\right)$$

Each outcome orthogonal to one of the possible input states

Requires no overlap, otherwise potential for confusion and getting “wrong result”

The quantum state cannot be interpreted statistically, Matthew F. Pusey, Jonathan Barrett & Terry Rudolph, arXiv:1111.3328v1
On the reality of the quantum state, Nature Physics 8, 475–478 (2012)

PBR Cont.

- Theorem holds in presence of imperfections and noise
- Can generalize to any pair of non-orthogonal quantum states
- Hence any underlying $\mu_\psi(\lambda)$ must be disjoint for all pairs of wavefunctions
- Hence different wavefunctions constitute distinct physical properties, are ontic
- Dropping “Preparation Independence” allows epistemic interpretation that matched QM

The quantum state can be interpreted statistically, P. G. Lewis, D. Jennings, J. Barrett, T. Rudolph, arXiv:1201.6554v1
Distinct Quantum States Can Be Compatible with a Single State of Reality, Phys. Rev. Lett. 109, 150404 (2012)

Undiscussed

- Hardy's Paradox
- Leggett Inequalities
- Leggett-Garg Inequalities
- Multi-partite non-locality
- Uncertainty bounds
- Generalized probability theories
- Decoherence Programme
- "Reasonable Axioms" implying QM
- Relativistic QM
- QM and Gravity
- Etc…

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